the effect per degree of the former being greater than the latter. The shortest latency for the heated frog muscle occurs at 29—30·5°, and closely precedes rigor.

Muscles maintained for a considerable time at a given elevation or depression of temperature, preserve a constant latency.

Up to the present the gastrocnemius of the frog has been the only muscle upon which we have experimented, and our chief object in making this communication is to call attention to the changes in the duration of the latency brought about by varying conditions of stimulation. We hope to continue our observation by making similar experiments on different muscles of various animals, and in a future paper to enter more fully upon the changes occurring in the other phases of the contraction, for which purpose further use will be made of the tables we have the honour of laying before the Royal Society.

"Formulæ for sn $8u$, cn $8u$, dn $8u$, in terms of sn u." By ERNEST H. GLAISHER, B.A., Trinity College, Cambridge. Communicated by J. W. L. GLAISHER, M.A., F.R.S. Received and Read June 16, 1881.

§ 1. In Grunert's "Archiv der Mathematik und Physik," vol. xxxvi (1861), pp. 125–176, Baehr has given the formulæ for sn nu, cn nu, dn nu, in terms of sn u for the cases $n=2, 3, 4, 5, 6, 7$. These expressions are reproduced by Cayley in a tabular form in his "Treatise on Elliptic Functions," Art. 109 (pp. 80–85).

The object of the present paper is to give the corresponding formulæ in the case of $n=8$. These were deduced from the formulæ for the case $n=4$ in the following manner.

We have

$$\operatorname{sn} 8u=\frac{2\operatorname{sn} 4u \operatorname{cn} 4u \operatorname{dn} 4u}{1-k^2 \operatorname{sn}^2 4u},$$

$$\operatorname{cn} 8u=\frac{1-2\operatorname{sn}^2 4u+k^2\operatorname{sn}^4 4u}{1-k^2\operatorname{sn}^2 4u},$$

$$\operatorname{dn} 8u=\frac{1-2k^2\operatorname{sn}^2 4u+k^2\operatorname{sn}^4 4u}{1-k^2\operatorname{sn}^2 4u},$$

and therefore, denoting the numerators of sn $4u$, cn $4u$, dn $4u$, and their common denominator by P, Q, R, S respectively, so that

$$\operatorname{sn} 4u=\frac{\mathrm{P}}{\mathrm{S}}, \qquad \operatorname{cn} 4u=\frac{\mathrm{Q}}{\mathrm{S}}, \qquad \operatorname{dn} 4u=\frac{\mathrm{R}}{\mathrm{S}},$$

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we find

$$\mathrm{sn}\,8u = \frac{2\mathrm{PQRS}}{\mathrm{S}^4 - k^2\mathrm{P}^4},$$

$$\mathrm{cn}\,8u = \frac{\mathrm{S}^4 - 2\mathrm{P}^2\mathrm{S}^2 + k^2\mathrm{P}^4}{\mathrm{S}^4 - k^2\mathrm{P}^4}$$

$$\mathrm{dn}\,8u = \frac{\mathrm{S}^4 - 2k^2\mathrm{P}^2\mathrm{S}^2 + k^2\mathrm{P}^4}{\mathrm{S}^4 - k^2\mathrm{P}^4}.$$

§ 2. The numerators of sn $8u$, cn $8u$, dn $8u$, and their common denominator, may therefore be deduced by combining linearly the four expressions PQRS, P^4, S^4, P^2, S^2, where

$$\mathrm{P} = x\sqrt{(1-x^2)}\sqrt{(1-k^2x^2)} \times \{4-(8+8k^2)x^2+20k^2x^4-20k^4x^8+(8k^4+8k^6)x^{10}-4k^6x^{12}\},$$

$$\mathrm{Q} = 1-8x^2+(8+20k^2)x^4-(24k^2+32k^4)x^6+(54k^4+16k^6)x^8 -(24k^4+32k^6)x^{10}+(8k^4+20k^6)x^{12}-8k^6x^{14}+k^8x^{16},$$

$$\mathrm{R} = 1-8k^2x^2+(20k^2+8k^4)x^4-(32k^2+24k^4)x^6+(16k^2+54k^4)x^8 -(32k^4+24k^6)x^{10}+(20k^6+8k^8)x^{12}-8k^8x^{14}+k^8x^{16},$$

$$\mathrm{S} = 1-20k^2x^4+(32k^2+32k^4)x^6-(16k^2+58k^4+16k^6)x^8 +(32k^4+32k^6)x^{10}-20k^6x^{12}+k^8x^{16},$$

and x denotes, as throughout this paper, sn u.

The values of PQRS, P^4, S^4, $\mathrm{P}^2\mathrm{S}^2$ were calculated in the following manner:

The squares P^2, S^2, and the products PS and QR, were first formed. Then P^2, S^2 were multiplied together, and the square of PS was formed: the agreement of these two results verified the values of P^2, S^2, $\mathrm{P}^2\mathrm{S}^2$. The expressions for P^4 and S^4 were then obtained by squaring P^2 and S^2; these calculations being performed in duplicate. To obtain PQRS the expressions for PS and QR were multiplied together; and as a verification the product PQ was formed, and this was multiplied successively by S and R.

§ 3. The resulting formulæ are shown in the following tables, in which the mode of arrangement is almost obvious: thus, for example, the numerator of sn $8u$

$$
\begin{aligned}
&= x\sqrt{\{(1-x^2)(1-k^2x^2)\}} \times \\
&\quad \{8 \\
&\quad -(80+80k^2)x^2 \\
&\quad +(192+968k^2+192k^4)x^4 \\
&\quad -(128+2496k^2+2496k^4+128k^6)x^6 \\
&\quad +(1728k^2-7416k^4+1728k^6)x^8 \\
&\quad +(87984k^4+87984k^6)x^{10} \\
&\quad \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
&\quad +(80k^{28}+80k^{30})x^{58} \\
&\quad -8k^{30}x^{60}\}
\end{aligned}
$$

Numerator of $\text{sn}\,8u = x\sqrt{\{(1-x^2)(1-k^2x^2)\}} \times$

	$k^0.$	$k^2.$	$k^4.$	$k^6.$	$k^8.$	$k^{10}.$	$k^{12}.$	$k^{14}.$
$x^0 k^0$	+ 8							
$x^2 k^0$	− 80	− 80						
$x^4 k^0$	+ 192	+ 968	+ 192					
$x^6 k^0$	− 128	− 2496	− 2496	− 128				
$x^8 k^2$	+ 1728	− 7416	+ 1728					
$x^{10} k^4$	+ 87984	+ 87984						
$x^{12} k^4$	− 274176	− 645048	− 274176					
$x^{14} k^4$	+ 496128	+ 2049792	+ 2049792	+ 496128				
$x^{16} k^4$	− 583680	− 3954432	− 6962136	− 3954432	− 583680			
$x^{18} k^4$	+ 430080	+ 4915200	+ 14388720	+ 14388720	+ 4915200	+ 430080		
$x^{20} k^4$	− 180224	− 3794944	− 18971200	− 31859224	− 18971200	− 3794944	− 180224	
$x^{22} k^4$	+ 32768	+ 1654784	+ 15392128	+ 44633920	+ 44633920	+ 15392128	+ 1654784	+ 32768
$x^{24} k^6$	− 311296	− 6998016	− 38113344	− 65268568	− 38113344	− 6998016	− 311296	
$x^{26} k^8$	+ 1363968	+ 18087936	+ 55747056	+ 55747056	+ 18087936	+ 1363968		
$x^{28} k^{10}$	− 3655680	− 23083008	− 40128024	− 23083008	− 3655680			
x^{30}	0							
$x^{32} k^{12}$	+ 3655680	+ 23083008	+ 40128024	+ 23083008	+ 3655680			
$x^{34} k^{12}$	− 1363968	− 18087936	− 55747056	− 55747056	− 18087936	− 1363968		
$x^{36} k^{12}$	+ 311296	+ 6998016	+ 38113344	+ 65268568	+ 38113344	+ 6998016	+ 311296	
$x^{38} k^{12}$	− 32768	− 1654784	− 15392128	− 44633920	− 44633920	− 15392128	− 1654784	− 32768
$x^{40} k^{14}$	+ 180224	+ 3794944	+ 18971200	+ 31859224	+ 18971200	+ 3794944	+ 180224	
$x^{42} k^{16}$	− 430080	− 4915200	− 14388720	− 14388720	− 4915200	− 430080		
$x^{44} k^{18}$	+ 583680	+ 3954432	+ 6962136	+ 3954432	+ 583680			
$x^{46} k^{20}$	− 496128	− 2049792	− 2049792	− 496128				
$x^{48} k^{22}$	+ 274176	+ 645048	+ 274176					
$x^{50} k^{24}$	− 87984	− 87984						
$x^{52} k^{24}$	− 1728	+ 7416	− 1728					
$x^{54} k^{24}$	+ 128	+ 2496	+ 2496	+ 128				
$x^{56} k^{26}$	− 192	− 968	− 192					
$x^{58} k^{28}$	+ 80	+ 80						
$x^{60} k^{30}$	− 8							

Numerator of cn $8u$=

	k^0.	k^2.	k^4.	k^6.	k^8.	k^{10}.	k^{12}.
$x^0 k^0$	+ 1						
$x^2 k^0$	− 32						
$x^4 k^0$	+ 160	+ 336					
$x^6 k^0$	− 256	− 2016	− 2688				
$x^8 k^0$	+ 128	+ 3456	+ 21816	+ 10560			
$x^{10} k^2$	− 1792	− 77088	− 99968	− 22528			
$x^{12} k^4$	+ 209888	+ 435440	+ 234240	+ 26624			
$x^{14} k^4$	− 470016	− 1448672	− 1135872	− 294912	− 16384		
$x^{16} k^4$	+ 744192	+ 3642624	+ 4407964	+ 1529472	+ 189952	+ 4096	
$x^{18} k^4$	− 778240	− 6206464	− 12575904	− 7406848	− 1032192	− 49152	
$x^{20} k^4$	+ 516096	+ 6885376	+ 23453856	+ 25350480	+ 8029952	+ 276480	
$x^{22} k^4$	− 196608	− 4792320	− 27864320	− 53121888	− 35957120	− 7092224	
$x^{24} k^4$	+ 32768	+ 1900544	+ 20467584	+ 68399744	+ 87045768	+ 41956032	+ 5990400
$x^{26} k^6$	− 327680	− 8478720	− 53349120	− 124358560	− 115922304	− 40808448	− 4128768
$x^{28} k^8$	+ 1515520	+ 23162880	+ 107670240	+ 184895600	+ 122807808	+ 29483008	+ 1900544
$x^{30} k^{10}$	− 4300800	− 55238656	− 183281760	− 214341120	− 93880320	− 14155776	− 524288
$x^{32} k^{12}$	+ 16755200	+ 118052352	+ 239133510	+ 175676160	+ 47342592	+ 4055040	+ 65536
$x^{34} k^{12}$	− 4300800	− 55238656	− 183281760	− 214341120	− 93880320	− 14155776	− 524288
$x^{36} k^{12}$	+ 1515520	+ 23162880	+ 107670240	+ 184895600	+ 122807808	+ 29483008	+ 1900544
$x^{38} k^{12}$	− 327680	− 8478720	− 53349120	− 124358560	− 115922304	− 40808448	− 4128768
$x^{40} k^{12}$	+ 32768	+ 1900544	+ 20467584	+ 68399744	+ 87045768	+ 41956032	+ 5990400
$x^{42} k^{14}$	− 196608	− 4792320	− 27864320	− 53121888	− 35957120	− 7092224	
$x^{44} k^{16}$	+ 516096	+ 6885376	+ 23453856	+ 25350480	+ 8029952	+ 276480	
$x^{46} k^{18}$	− 778240	− 6206464	− 12575904	− 7406848	− 1032192	− 49152	
$x^{48} k^{20}$	+ 744192	+ 3642624	+ 4407964	+ 1529472	+ 189952	+ 4096	
$x^{50} k^{22}$	− 470016	− 1448672	− 1135872	− 294912	− 16384		
$x^{52} k^{24}$	+ 209888	+ 435440	+ 234240	+ 26624			
$x^{54} k^{24}$	− 1792	− 77088	− 99968	− 22528			
$x^{56} k^{24}$	+ 128	+ 3456	+ 21816	+ 10560			
$x^{58} k^{26}$	− 256	− 2016	− 2688				
$x^{60} k^{28}$	+ 160	+ 336					
$x^{62} k^{30}$	− 32						
$x^{64} k^{32}$	+ 1						

Numerator of $\operatorname{dn} 8u =$

	k^0.	k^4.	k^6.	k^8.	k^{10}.	k^{12}.	k^{14}.
$x^0 k^0$	+ 1						
$x^2 k^2$	− 32						
$x^4 k^2$	+ 336	+ 160					
$x^6 k^2$	− 2688	− 2016	− 256				
$x^8 k^2$	+ 10560	+ 21816	+ 3456	+ 128			
$x^{10} k^2$	− 22528	− 99968	− 77088	− 1792			
$x^{12} k^2$	+ 26624	+ 234240	+ 435440	+ 209888			
$x^{14} k^2$	− 16384	− 294912	− 1135872	− 1448672	− 470016		
$x^{16} k^2$	+ 4096	+ 189952	+ 1529472	+ 4407964	+ 3642624	+ 744192	
$x^{18} k^4$	− 49152	− 1032192	− 7406848	− 12575904	− 6206464	− 778240	
$x^{20} k^6$	+ 276480	+ 8029952	+ 25350480	+ 23453856	+ 6885376	+ 516096	
$x^{22} k^8$	− 7092224	− 35957120	− 53121888	− 27864320	− 4792320	− 196608	
$x^{24} k^8$	+ 5990400	+ 41956032	+ 87045768	+ 68399744	+ 20467584	+ 1900544	+ 32768
$x^{26} k^8$	− 4128768	− 40808448	− 115922304	− 124358560	− 53349120	− 8478720	− 327680
$x^{28} k^8$	+ 1900544	+ 29483008	+ 122807808	+ 184895600	+ 107670240	+ 23162880	+ 1515520
$x^{30} k^8$	− 524288	− 14155776	− 93880320	− 214341120	− 183281760	− 55238656	− 4300800
$x^{32} k^8$	+ 65536	+ 4055040	+ 47342592	+ 175676160	+ 239133510	+ 118052352	+ 16755200
$x^{34} k^{10}$	− 524288	− 14155776	− 93880320	− 214341120	− 183281760	− 55238656	− 4300800
$x^{36} k^{12}$	+ 1900544	+ 29483008	+ 122807808	+ 184895600	+ 107670240	+ 23162880	+ 1515520
$x^{38} k^{14}$	− 4128768	− 40808448	− 115922304	− 124358560	− 53349120	− 8478720	− 327680
$x^{40} k^{16}$	+ 5990400	+ 41956032	+ 87045768	+ 68399744	+ 20467584	+ 1900544	+ 32768
$x^{42} k^{18}$	− 7092224	− 35957120	− 53121888	− 27864320	− 4792320	− 196608	
$x^{44} k^{18}$	+ 276480	+ 8029952	+ 25350480	+ 23453856	+ 6885376	+ 516096	
$x^{46} k^{18}$	− 49152	− 1032192	− 7406848	− 12575904	− 6206464	− 778240	
$x^{48} k^{18}$	+ 4096	+ 189952	+ 1529472	+ 4407964	+ 3642624	+ 744192	
$x^{50} k^{20}$	− 16384	− 294912	− 1135872	− 1448672	− 470016		
$x^{52} k^{22}$	+ 26624	+ 234240	+ 435440	+ 209888			
$x^{54} k^{24}$	− 22528	− 99968	− 77088	− 1792			
$x^{56} k^{26}$	+ 10560	+ 21816	+ 3456	+ 128			
$x^{58} k^{28}$	− 2688	− 2016	− 256				
$x^{60} k^{30}$	+ 336	+ 160					
$x^{62} k^{32}$	− 32						
$x^{64} k^{32}$	+ 1						

The Common Denominator=

	k^0.	k^2.	k^4.	k^6.	k^8.	k^{10}.	k^{12}.	k^{14}.	k^{16}.
$x^0 k^0$	+ 1								
x^2	0								
$x^4 k^2$	− 336								
$x^6 k^2$	+ 2688	+ 2688							
$x^8 k^2$	− 10560	− 24456	− 10560						
$x^{10} k^2$	+ 22528	+ 105600	+ 105600	+ 22528					
$x^{12} k^2$	− 26624	− 240896	− 498160	− 240896	− 26624				
$x^{14} k^2$	+ 16384	+ 299008	+ 1212160	+ 1212160	+ 299008	+ 16384			
$x^{16} k^2$	− 4096	− 190976	− 1577600	− 2859236	− 1577600	− 190976	− 4096		
$x^{18} k^4$	+ 49152	+ 1044480	+ 2750208	+ 2750208	+ 1044480	+ 49152			
$x^{20} k^6$	− 276480	+ 968448	+ 3136944	+ 968448	− 276480				
$x^{22} k^8$	− 5154816	− 20678784	− 20678784	− 5154816					
$x^{24} k^8$	+ 5990400	+ 38714688	+ 68067144	+ 38714688	+ 5990400				
$x^{26} k^8$	− 4128768	− 41840640	− 118173312	− 118173312	− 41840640	− 4128768			
$x^{28} k^8$	+ 1900544	+ 29958144	+ 130475520	+ 208436624	+ 130475520	+ 29958144	+ 1900544		
$x^{30} k^8$	− 524288	− 14286848	− 97501184	− 240084480	− 240084480	− 97501184	− 14286848	− 524288	
$x^{32} k^8$	+ 65536	+ 4071424	+ 48366592	+ 188153088	+ 290931270	+ 188153088	+ 48366592	+ 4071424	+ 65536
$x^{34} k^{10}$	− 524288	− 14286848	− 97501184	− 240084480	− 240084480	− 97501184	− 14286848	− 524288	
$x^{36} k^{12}$	+ 1900544	+ 29958144	+ 130475520	+ 208436624	+ 130475520	+ 29958144	+ 1900544		
$x^{38} k^{14}$	− 4128768	− 41840640	− 118173312	− 118173312	− 41840640	− 4128768			
$x^{40} k^{16}$	+ 5990400	+ 38714688	+ 68067144	+ 38714688	+ 5990400				
$x^{42} k^{18}$	− 5154816	− 20678784	− 20678784	− 5154816					
$x^{44} k^{18}$	− 276480	+ 968448	+ 3136944	+ 968448	− 276480				
$x^{46} k^{18}$	+ 49152	+ 1044480	+ 2750208	+ 2750208	+ 1044480	+ 49152			
$x^{48} k^{18}$	− 4096	− 190976	− 1577600	− 2859236	− 1577600	− 190976	− 4096		
$x^{50} k^{20}$	+ 16384	+ 299008	+ 1212160	+ 1212160	+ 299008	+ 16384			
$x^{52} k^{22}$	− 26624	− 240896	− 498160	− 240896	− 26624				
$x^{54} k^{24}$	+ 22528	+ 105600	+ 105600	+ 22528					
$x^{56} k^{26}$	− 10560	− 24456	− 10560						
$x^{58} k^{28}$	+ 2688	+ 2688							
$x^{60} k^{30}$	− 336								
x^{62}	0								
$x^{64} k^{32}$	+ 1								

§ 4. If k be put equal to unity the cn and dn become identical, and the formulæ, as is well known, assume the simple forms

$$\operatorname{sn} nu=\frac{(1+x)^n-(1-x)^n}{(1+x)^n+(1-x)^n} \quad . \quad . \quad . \quad . \quad . \quad (k=1),$$

$$\operatorname{cn} nu=\frac{2(1-x^2)^{\frac{1}{2}n}}{(1+x)^n+(1-x)^n} \quad . \quad . \quad . \quad . \quad . \quad (k=1).$$

Putting $n=8$ in these formulæ, we have

$$\operatorname{sn} 8u=\frac{8(x+7x^3+7x^5+x^7)}{1+28x^2+70x^4+28x^6+x^8},$$

$$\operatorname{cn} 8u=\frac{1-4x^2+6x^4-4x^6+x^8}{1+28x^2+70x^4+28x^6+x^8}.$$

When k is put equal to 1 the formulæ in § 3 should reduce to these expressions: and we thus obtain an important verification of their accuracy.

Since the denominator of sn $8u$, cn $8u$, dn $8u$ is of the order 64 in x, it is evident that a factor of the order 56 is common to the numerator and denominator of these expressions when k is put equal to unity. By putting $k=1$ in the formulæ of § 3 and dividing the resulting expression for the numerator of sn $8u$ by $1+7x^2+7x^4+x^6$ it is found that this factor is $(1-x^2)^{28}$, as it should be. And it was verified by division that the expressions for the numerator of cn $8u$ and the common denominator were equal to this factor multiplied by $1-4x^2+6x^4-4x^6+x^8$ and $1+28x^2+70x^4+28x^6+x^8$ respectively.

§ 5. The expressions P^2, S^2, P^4, S^4 are respectively the numerators and denominators of $\operatorname{sn}^2 u$ and of $\operatorname{sn}^4 u$, and it seems worth while to place on record their values, which are as follows:

$P^2 =$

	k^0.	k^2.	k^4.	k^6.
$x^2 k^0$	+ 16			
$x^4 k^0$	− 80	− 80		
$x^6 k^0$	+ 128	+ 432	+ 128	
$x^8 k^0$	− 64	− 736	− 736	− 64
$x^{10} k^2$	+ 384	+ 1168	+ 384	
$x^{12} k^4$	− 176	− 176		
$x^{14} k^4$	− 512	− 1616	− 512	
$x^{16} k^4$	+ 128	+ 1984	+ 1984	+ 128
$x^{18} k^6$	− 512	− 1616	− 512	
$x^{20} k^8$	− 176	− 176		
$x^{22} k^8$	+ 384	+ 1168	+ 384	
$x^{24} k^8$	− 64	− 736	− 736	+ 64
$x^{26} k^{10}$	+ 128	+ 432	+ 128	
$x^{28} k^{12}$	− 80	− 80		
$x^{30} k^{14}$	+ 16			

$S^2 =$

	k^0.	k^2.	k^4.	k^6.	k^8.
$x^0 k^0$	+ 1				
x^2	0				
$x^4 k^2$	− 40				
$x^6 k^2$	+ 64	+ 64			
$x^8 k^2$	− 32	+ 284	− 32		
$x^{10} k^4$	− 1216	− 1216			
$x^{12} k^4$	+ 1664	+ 4328	+ 1664		
$x^{14} k^4$	− 1024	− 6016	− 6016	− 1024	
$x^{16} k^4$	+ 256	+ 3904	+ 8774	+ 3904	+ 256
$x^{18} k^6$	− 1024	− 6016	− 6016	− 1024	
$x^{20} k^8$	+ 1664	+ 4328	+ 1664		
$x^{22} k^{10}$	− 1216	− 1216			
$x^{24} k^{10}$	− 32	+ 284	− 32		
$x^{26} k^{12}$	+ 64	+ 64			
$x^{28} k^{14}$	− 40				
x^{30}	0				
$x^{32} k^{16}$	+ 1				

$P^4 =$

	k^0.	k^2.	k^4.	k^6	k^8.	k^{10}.	k^{12}.
$x^4 k^0$	+ 256						
$x^6 k^0$	− 2560	− 2560					
$x^8 k^0$	+ 10496	+ 26624	+ 10496				
$x^{10} k^0$	− 22528	− 113152	− 113152	− 22528			
$x^{12} k^0$	+ 26624	+ 250880	+ 492288	+ 250880	+ 26624		
$x^{14} k^0$	− 16384	− 305152	− 1094656	− 1094656	− 305152	− 16384	
$x^{16} k^0$	+ 4096	+ 192512	+ 1278464	+ 2301952	+ 1278464	+ 192512	+ 4096
$x^{18} k^2$	− 49152	− 673792	− 2126848	− 2126848	− 673792	− 49152	
$x^{20} k^4$	+ 18432	− 31744	− 167680	− 31744	+ 18432		
$x^{22} k^4$	+ 98304	+ 1525760	+ 5241344	+ 5241344	+ 1525760	+ 98304	
$x^{24} k^4$	− 16384	− 966656	− 6584576	− 11937792	− 6584576	− 966656	− 16384
$x^{26} k^6$	+ 163840	+ 2854912	+ 9734656	+ 9734656	+ 2854912	+ 163840	
$x^{28} k^8$	− 45056	− 481280	− 849152	− 481280	− 45056		
$x^{30} k^8$	− 196608	− 3379200	− 11638784	− 11638784	− 3379200	− 196608	
$x^{32} k^8$	+ 24576	+ 1548288	+ 11115520	+ 20267008	+ 11115520	+ 1548288	+ 24576
$x^{34} k^{10}$	− 196608	− 3379200	− 11638784	− 11638784	− 3379200	− 196608	
$x^{36} k^{12}$	− 45056	− 481280	− 849152	− 481280	− 45056		
$x^{38} k^{12}$	+ 163840	+ 2854912	+ 9734656	+ 9734656	+ 2854912	+ 163840	
$x^{40} k^{12}$	− 16384	− 966656	− 6584576	− 11937792	− 6584576	− 966656	− 16384
$x^{42} k^{14}$	+ 98304	+ 1525760	+ 5241344	+ 5241344	+ 1525760	+ 98304	
$x^{44} k^{16}$	+ 18432	− 31744	− 167680	− 31744	+ 18432		
$x^{46} k^{16}$	− 49152	− 673792	− 2126848	− 2126848	− 673792	− 49152	
$x^{48} k^{16}$	+ 4096	+ 192512	+ 1278464	+ 2301952	+ 1278464	+ 192512	+ 4096
$x^{50} k^{18}$	− 16384	− 305152	− 1094656	− 1094656	− 305152	− 16384	
$x^{52} k^{20}$	+ 26624	+ 250880	+ 492288	+ 250880	+ 26624		
$x^{54} k^{22}$	− 22528	− 113152	− 113152	− 22528			
$x^{56} k^{24}$	+ 10496	+ 26624	+ 10496				
$x^{58} k^{26}$	− 2560	− 2560					
$x^{60} k^{28}$	+ 256						

$S^4 =$

	k^0.	k^2.	k^4.	k^6.	k^8.	k^{10}.	k^{12}.	k^{14}.	k^{16}.
$x^0 k^0$	+ 1								
x^2									
$x^4 k^2$	− 80								
$x^6 k^2$	+ 128	+ 128							
$x^8 k^2$	− 64	+ 2168	− 64						
$x^1 k^4$	− 7552	− 7552							
$x^1 k$	+ 9984	− 5872	+ 9984						
$x^{14} k^4$	− 6144	+ 117504	+ 117504	− 6144					
$x^{16} k^4$	+ 1536	− 299136	− 557284	− 299136	+ 1536				
$x^{18} k^6$	+ 370688	+ 623360	+ 623360	+ 370688					
$x^{20} k^6$	− 258048	+ 936704	+ 2969264	+ 936704	− 258048				
$x^{22} k^6$	+ 98304	− 3629056	− 15437440	− 15437440	− 3629056	+ 98304			
$x^{24} k^6$	− 16384	+ 5023744	+ 32130112	+ 56129352	+ 32130112	+ 5023744	− 16384		
$x^{26} k^8$	− 3964928	− 38985728	− 108438656	− 108438656	− 38985728	− 3964928			
$x^{28} k^8$	+ 1900544	+ 29913088	+ 129994240	+ 207587472	+ 129994240	+ 29913088	+ 1900544		
$x^{30} k^8$	− 524288	− 14483456	− 100880384	− 251723264	− 251723264	− 100880384	− 14483456	− 524288	
$x^{32} k^8$	+ 65536	+ 4096000	+ 49914880	+ 199268608	+ 311198278	+ 199268608	+ 49914880	+ 4096000	+ 65536
$x^{34} k^{10}$	− 524288	− 14483456	− 100880384	− 251723264	− 251723264	− 100880384	− 14483456	− 524288	
$x^{36} k^{12}$	+ 1900544	+ 29913088	+ 129994240	+ 207587472	+ 129994240	+ 29913088	+ 1900544		
$x^{38} k^{14}$	− 3964928	− 38985728	− 108438656	− 108438656	− 38985728	− 3964928			
$x^{40} k^{14}$	− 16384	+ 5023744	+ 32130112	+ 56129352	+ 32130112	+ 5023744	− 16384		
$x^{42} k^{16}$	+ 98304	− 3629056	− 15437440	− 15437440	− 3629056	+ 98304			
$x^{44} k^{18}$	− 258048	+ 936704	+ 2969264	+ 936704	− 258048				
$x^{46} k^{20}$	+ 370688	+ 623360	+ 623360	+ 370688					
$x^{48} k^{20}$	+ 1536	− 299136	− 557284	− 299136	+ 1536				
$x^{50} k^{22}$	− 6144	+ 117504	+ 117504	− 6144					
$x^{52} k^{24}$	+ 9984	− 5872	+ 9984						
$x^{54} k^{26}$	− 7552	− 7552							
$x^{56} k^{26}$	− 64	+ 2168	− 64						
$x^{58} k^{28}$	+ 128	+ 128							
$x^{60} k^{30}$	− 80								
x^{62}									
$x^{64} k^3$	+ 1								